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# Earth Radius/Kilometer Conversion Factor for the Lunar Ephemeris

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## I Introduction

IN precision simulations of lunar and interplanetary probe trajectories, the equations of motion contain terms that require a knowledge of the position of the moon at any time during flight Furthermore, for calculation of the probe's position and velocity relative to the moon, *both* the position and velocity of the moon are required Common practice is to obtain the position of the moon from the Lunar Ephemeris[1] and subsequently develop the velocity by numerical differentiation of the position

A difficulty arises in that the lunar coordinates given in the Ephemeris use the earth radius as a unit of length, whereas, for practical reasons, a laboratory unit of length such as the kilometer is employed as the basic unit of measure in trajectory calculations The problem is then one of determining a conversion (or scale) factor to convert the lunar coordinates from earth radii to kilometers

At first impulse, one would choose the best available value of the earth equatorial radius, as expressed in kilometers, and use this as the conversion factor However, to do so would be incorrect Rather, the conversion factor must be computed from a relationship that is a function of the moon's mean motion and the gravitational constants of the earth and moon

It is the purpose of this paper to develop this relationship and give a value for the earth radius/kilometer conversion factor for the Lunar Ephemeris

## II Analysis

De Sitter[2] defines the sine of the mean equatorial lunar parallax as

$$\sin\pi_{☾} = b/a \tag{1}$$

where $b$ is the equatorial radius of the earth and $a$ is the "constant of the moon's variation orbit" defined in Brown's theory [3, 4] The constant $\alpha$ is defined by the relation

$$n^2\alpha^3 = GM_E + GM_M \tag{2}$$

where $G$ is the universal gravitational constant, $M_E$ and $M_M$ are the masses of the earth and moon, and $n$ is the moon's mean sidereal motion, which is taken as a fundamental in variant, the value of which, as used by Brown, is $n = 0\ 2661699563 \times 10^{-5}$ rad/sec Brown also gives a relation between $a$ and $\alpha$; it is

$$a/\alpha = 0\ 999093141975298 \tag{3}$$

Thus, it is seen that, if $GM_E + GM_M$ and $\sin\pi_{☾}$ are known, then $b$ can be calculated Brown gives

$$\sin\pi_{☾} = 3422''54 = 0\ 01659294212$$

Thus,

$$b/a = 0\ 01659294212$$

or

$$a = 60\ 2665876b \tag{4}$$

which establishes the relation between the mean lunar distance and earth radius as Brown sees it From Eq (3),

$$\alpha = 1\ 000907681a \tag{5}$$

Then

$$\alpha = 60\ 321290446 \tag{6}$$

After substituting (6) into (2), we obtain

$$b = 0\ 0165778946[(GM_E + GM_M)/n^2]^{1/3} \tag{7}$$

or finally

$$b = 86\ 315745(GM_E + GM_M)^{1/3} \tag{8}$$

## III Conclusion

The formula, Eq (8), is the relation mentioned earlier for computing the earth radius/kilometer conversion factor $b$, when $GM_E$ and $GM_M$ are given in kilometers cubed per seconds squared This relation is analogous to Kepler's third law and must be maintained If some other value of $b$ is used, the well-determined mean motion of the moon is not preserved

To calculate a value of $b$ from Eq (8), let $GM_E = 398603\ 2$ km$^3$/sec$^2$ [5] and note that $K_{ME}$, "the coefficient of the indirect acceleration of the moon on the earth," has been well determined from the orbit of Mariner II,[6] where

$$K_{ME} = GM_M/b^2 = 1\ 205116 \pm 0\ 000049 \times 10^{-4} \tag{9}$$

A cubic equation in $GM_M$ can be formed by using Eq (8) and (9) to eliminate $b$ and yield

$$GM_M = 4902\ 78 \pm 0\ 20 \text{ km}^3/\text{sec}^2$$

and thus give an earth-moon mass ratio of

$$\mu = 81\ 3015 \pm 0\ 0033$$

Subsequently, from Eq (9),

$$b = (GM_M/K_{ME})^{1/2} \tag{10}$$

or, finally, the numerical value of the earth radius/kilometer conversion factor is

$$b = 6378\ 3255 \text{ km}$$

Upon substituting this into Eq (4), we obtain the mean distance of the moon

$$a = 384399\ 9 \text{ km}$$

which compares favorably with Fischer's[7] value of 384,400 km and the radar-determined value of 384,400 2 km [8]

To conclude, it is important to realize that the value of earth radius $b$ is not the same as the actual radius of the earth; it is merely the conversion factor and is used *only* for scaling the Lunar Ephemeris from earth radii to kilometers The value of the actual radius of the earth $R_E$ is taken to be $R_E = 6378\ 165$ km [5]

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# Temperature and Velocity Profiles along a Vertical Hot Plate in a Compressible Fluid Considering the Effect of Buoyancy


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### Nomenclature

$T$ = absolute temperature
$c_p$ = specific heat at constant temperature
$c_v$ = specific heat at constant volume
$k$ = coefficient of heat conductivity
$\gamma$ = ratio of the specific heats
$P_r$ = Prandtl's number
$R/J$ = $c_p - c_v$
$J$ = joule equivalent
$g$ = acceleration due to gravity
$\beta$ = coefficient of cubical expansion
$\rho$ = density

***Subscripts***

$\omega$ = conditions at the wall
0 = conditions at $n = 0$
$\alpha$ = outside the boundary layer

### Introduction

THE original Blasius solution of the equations of the boundary-layer flow over a plate has been extended in various ways by different authors For instance, Chapman and Rubesin[1] have considered the flow and heat transfer in the boundary layer of a compressible fluid with zero pressuie gradient over a plate, taking viscous dissipation into consideration, with certain assumptions regarding $c_p, \mu$ and the Prandtl's number $P_r$ Pohlhausen,[2] on the other hand, solved the equation of liquid boundary-layer flow over a vertical hot plate taking buoyancy into consideration, but neglecting the frictional heat This problem was experimentally studied by Schmidt and Beckmann,[4] and Pohlhausen's theoretical results agreed fairly well with the experimental results The object of the present note is to show that the problem of the forementioned authors can be solved for a compressible flow under zero pressure gradient ignoring frictional heat, but taking buoyancy into consideration The Mises transformation[3] is found to be effective here as in the Karman-Tsien method

Consider a boundary layer in contact with a vertical hot plate and take the $x$ axis vertically upward along the hot plate The motion is steady and is supposed to be caused by the difference between the weight and the buoyancy in the gravitational field of the earth The small pressure gradient being neglected, the equations of motion in the boundary layer are

$$\rho u \frac{\partial u}{\partial x} + \rho v \frac{\partial u}{\partial y} = \frac{\partial}{\partial y}\left(\mu \frac{\partial u}{\partial y}\right) + \rho g \beta (T - T_\infty) \qquad (1)$$

$$\rho u \frac{\partial}{\partial x}(c_p T) + \rho v \frac{\partial}{\partial y}(c_p T) = \frac{\partial}{\partial y}\left(K \frac{\partial T}{\partial y}\right) \qquad (2)$$

$$\frac{\partial}{\partial x}(\rho u) + \frac{\partial}{\partial y}(\rho v) = 0 \qquad (3)$$

Frictional heat also has been neglected We shall solve the problem under the following assumptions:

1) $c_p$ = const

2) $P_r = c_p \mu / k$ = Prandtl's number

3) $\mu/\mu_\infty = cT/T_\infty$ when $c$ = const

On putting $\theta = (T - T_\infty)/(T_\infty - T_\infty)$, $T_\omega$ being the temperature on the wall, the equations are reduced to the form

$$\rho u \frac{\partial u}{\partial x} + \rho v \frac{\partial u}{\partial y} = \frac{\partial}{\partial y}\left(\mu \frac{\partial u}{\partial y}\right) + \rho a \theta \qquad (4)$$

$$\rho u \frac{\partial \theta}{\partial x} + \rho v \frac{\partial \theta}{\partial y} = \frac{1}{P_r}\frac{\partial}{\partial y}\left(\mu \frac{\partial \theta}{\partial y}\right) \qquad (5)$$

$$\frac{\partial}{\partial x}(\rho u) + \frac{\partial}{\partial y}(\rho v) = 0 \qquad (6)$$

when

$$a = g\beta(T_\omega - T_\infty)$$

We introduce a stream function $\psi$ by the equations

$$u = \frac{\rho_\infty}{\rho}\frac{\partial \psi}{\partial y} \qquad v = -\frac{\rho_\infty}{\rho}\frac{\partial \psi}{\partial x} \qquad (7)$$

Now effect a Mises transformation $(x,y) \rightarrow (x,\psi)$ according to

$$\left(\frac{\partial}{\partial y}\right) = \left(\frac{\rho u}{\rho_\infty}\right)\frac{\partial}{\partial \psi}$$

$$\left(\frac{\partial}{\partial_n}\right) = \left(\frac{\partial}{\partial x}\right)_\psi - \left(\frac{\rho v}{\rho_\infty}\right)\frac{\partial}{\partial \psi}$$

when Eqs (4) and (5) reduce to

$$u\frac{\partial u}{\partial x} = \frac{\mu_\infty}{\rho_\infty} u \frac{\partial}{\partial \psi}\left(cu \frac{\partial u}{\partial \psi}\right) + a\theta \qquad (8)$$

$$\frac{\partial \theta}{\partial x} = \frac{\mu_\infty}{P \rho_\infty}\frac{\partial}{\partial \psi}\left(cu\frac{\partial v}{\partial \psi}\right) \qquad (9)$$

We introduce dimensionless variables in the usual way by the substitutions $x^* = x/L$, $u^* = u/U$, $\mu^* = \mu/\mu_\infty$, $\psi^* =$

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